Warm-up!

f(x) = 3x + 1; $g(x) = \frac{x-1}{3}$

Find

1. $[f \circ g](x) =$ $3\left(\frac{x-1}{3}\right)+1 = x$
2. $[g \circ f](x) =$ $\frac{3x+1-1}{3} = x$

3. Find the inverse.

y = 2x - 3

$x = 2y - 3$

$x+3 = 2y$

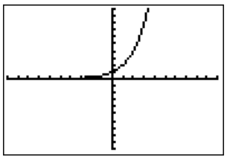
$\frac{x+3}{2} = y$

10-2 Logarithms and Logarithmic Functions

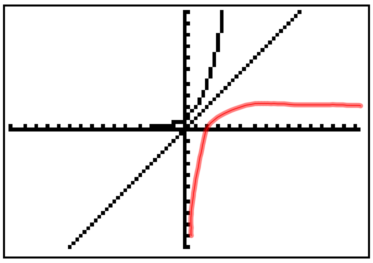
logarithm was invented by John Napier in 1614

$y = 2^x$

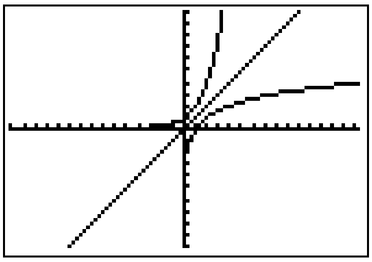
$x = 2^y$



Does this graph have an inverse?



Does this graph have an inverse?



The inverse of $y = 2^x$ is $x = 2^y$.

$x = 2^y$ is commonly expressed as $y = \log_2 x$

(graph both on Desmos)

Inverses of each other

$y = b^x$ $y = \log_b x$

What is the inverse?

$y = 10^x$ $y = \log_{10} x$

Characteristics of a Logarithmic Function

1. The function is continuous and one-to-one.
2. The domain is the set of all positive real numbers.
3. The *y*-axis is an asymptote of the graph.
4. The range is the set of all real numbers.
5. The graph contains the point (1, 0). That is, the *x*-intercept is 1.

Suppose b and x are positive, and $b \neq 1$, then, there is a number y such that:

$\log_b x = y$ iff $b^y = x$

(Used to convert between logarithmic and exponential form.)

$\log_b x = y$ iff $b^y = x$

Logarithmic Form	Exponential Form
$\log_2 16 = 4$	$2^4 = 16$
$\log_2 8 = 3$	$2^3 = 8$
$\log_2 1 = 0$	$2^0 = 1$
$\log_2 x = y$	$2^y = x$
$\log_b x = y$	$b^y = x$

$\log_b x = y$ iff $b^y = x$

Logarithmic Form	Exponential Form
$\log_{10} 1000 = 3$	$10^3 = 1000$
$\log_{16} 4 = .5$	$16^{1/2} = 4$
$\log_3 27 = 3$	$3^3 = 27$
$\log_9 81 = 2$	$9^2 = 81$
$\log_5 25 = 2$	$5^2 = 25$
$\log_3 243 = 5$	$3^5 = 243$

Evaluate a logarithmic expression.

ex
$\log_2 64$

6

$\log_2 64 = y$

$2^y = 64$

$2^y = 2^6$

6

ex
$\log_{25} 5$

$\frac{1}{2}$

$25^y = 5$

$5^{2y} = 5^1$

ex
$\log_{10} 0.1$

-1

$10^y = \frac{1}{10}$

$10^y = 10^{-1}$

Remember:
Two functions are inverses iff
$[f \circ g]\, x = x$ and $[g \circ f]\, x = x$

Inverses of each other

$y = b^x$ $\quad$ $y = \log_b x$

$f(x) = b^x$ $\quad$ $g(x) = \log_b x$

$b^{\log_b x} = x$ $\quad$ Properties of logs

$\log_b b^x = x$

ex
$\log_2 2^5$
5

ex
$\log_4 4^2$
2

ex
$\log_7 49$
2

ex
$3^{\log_3 5}$
5

ex
$8^{\log_8 10}$
10

DO:
1. $\log_{1/2} 32 = y$ $\left(\frac{1}{2}\right)^y = 32$ $2^{-y} = 2^5$ -5
2. $\log_9 27$ $9^y = 27$ $3^{2y} = 3^3$ $\boxed{\frac{3}{2}}$
3. $\log_5 125$ 3
4. $\log_8 4$ $8^y = 4$ $2^{3y} = 2^2$ $\frac{2}{3}$
5. $9^{\log_9 5}$ $9^{\log_9 5}$ 5
6. $\log_{\sqrt{3}} 9\sqrt{3}$ $\sqrt{3}^y = 9\sqrt{3}$
$3^{\frac{1}{2}y} = 3^2 \cdot 3^{\frac{1}{2}}$
$3^{\frac{1}{2}y} = 3^{2\frac{1}{2}}$
5

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